

ESTIMATING HEIGHTS

USING HAWAIIAN ANTHROPIC MEASUREMENTS AND TRIANGLES

BY RYAN GIRARD

How can something be measured indirectly?
 How can different methods be used to arrive at one answer?
 What are possible sources of errors when estimating?

HIGH SCHOOL OR COLLEGE LEVEL TRIGONOMETRY/MATH FOR
 ELEMENTARY TEACHERS

TIMEFRAME TWO OR MORE 60 MIN CLASS PERIODS
 (One day needs to be sunny, preferably at least 90 minutes before or after solar noon)

STANDARD BENCHMARKS AND VALUES

COMMON CORE STATE STANDARDS: MATH CONTENT

- HSG.SRT.B.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- HSG.SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- HSG.SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

COMMUNITY COLLEGE (TRIGONOMETRY): Apply quantitative methods appropriately; analyze real-life

situations using numeric, graphical, and symbolic models, and verbally explain these models; and recognize the impact of mathematics on the sciences, society, and everyday life.

GENERAL LEARNER OUTCOMES:

- Self-directed Learner
- Community Contributor
- Complex Thinker
- Effective Communicator

NĀ HOPENA A'O: Strengthened Sense of Responsibility

NĀ HONUA MAULI OLA: 'Ike Na'auao (Intellectual Pathway)

ENDURING UNDERSTANDING

- Sometimes it is not possible to find exact values and estimation techniques must be used;
- One's hands can be used for approximate measurements;
- Estimating is a legitimate approximation tool.

CRITICAL SKILLS AND CONCEPTS

Participants will be able to...

- Identify and pronounce the Hawaiian names for anthropic hand measures;
- Use ratios and/or trigonometry to determine heights using similar triangles;
- Use trigonometry and a calculator to create a chart for hand-to-degree calibration and conversion.

AUTHENTIC PERFORMANCE TASK:

- Students will estimate the height of various objects that they cannot measure directly (flag or telephone pole, tree, building, etc.).
- Students will gather individual results and use them to come to a group consensus.

AUTHENTIC AUDIENCE:

Friends and/or family members in a context where direct measurement is not possible.

OTHER EVIDENCE:

- Students will complete the Calibration and Estimation Worksheets.
- They will also estimate the height of a few objects that they can measure (small tree, sign post, fence) to see how close their estimation is to the actual height.





LEARNING PLAN

DAY ONE: INTRODUCTION OF SELECT HAWAIIAN ANTHROPIC UNITS & CALIBRATION

MATERIALS: Tape measures (at least one tape measure that is 100+ ft. is preferable but not necessary).

Anthropic Units

1. Have students trace out their hand.



2. Have students indicate the Mākahi, Mālua, Mākolu, and Māhā measurements as described below.

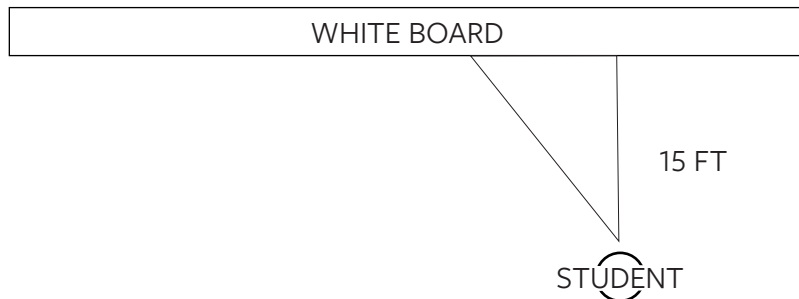
Anakahi kino (Hawaiian anthropic measurements)

Mākahi	One (1) finger's width
Mālua	Two (2) fingers' width
Mākolu	Three (3) fingers' width
Māhā	Four (4) fingers' width
Kīko'o	the diagonal distance from tip of pointer finger to tip of thumb (on same hand)
Pī'ā	from tip of middle finger to heel of hand (or bottom of palm)
Kaulua	from tip of thumb to tip of pinky finger outstretched (i.e., a span or "shaka")

3. Continue by discussing the Kīkoʻo, Pīʻā, and Kaulua.
Have students repeat each word as you pronounce it.

CREATING THE HAND-TO-DEGREE CALIBRATION CHART

Students will be creating a calibration chart by standing 15 feet away from a whiteboard.



4. Students will first use trigonometry to determine the necessary distance across the whiteboard. Divide students into groups. One group can do degrees 1-5, another can do 6-10, etc.

The following equation can be used to determine the measurement:

$$\tan(\theta) = \frac{Y}{15}$$

Where θ = the angle measure (in degrees) and will range from 1 to 25 degrees. Y = the distance across the whiteboard, and 15 represents the distance (in feet) that the student is standing from the whiteboard. It is recommended that students convert their answers into either the nearest inch or __ft __in format (see example).

	Total	Total			Approx.	
Degree	Wall Side (ft)	Inches	ft.	in.	ft.	in.
1	0.26	3.14	0	3.14	0	3
2	0.52	6.29	0	6.29	0	6
3	0.79	9.43	0	9.43	0	9
4	1.05	12.59	1	0.59	1	1

(See blank worksheet at end of lesson)

5. As a class (or in groups if multiple calibration sites will be used), find the average height between the ground and an arm that is held parallel to the ground.



- Students will then measure and mark the appropriate calibration distances found in step 4 on the whiteboard. The measurements should take place at the same height about the ground that students determined in step 5.
- Standing 15 ft back from the white board, have students complete at least the following conversions.

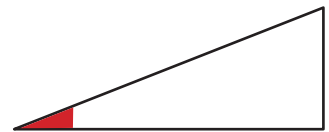
Māhā = _____ degrees
 Kaulua = _____ degrees

- As a class, discuss the differences. Was there a most common (mode) degree for the Māhā or the Kaulua?
- If time permits, discuss what will happen during the second class when students will get their shadow and distance measurements. Also discuss how the hand measurements and shadows will be used to estimate the heights of objects.



Method 1) Students will collect the following information:

Height of Person = _____ (ft. and in.) = _____ (in.)
 Shadow of Person = _____ (ft. and in.) = _____ (in.)
 Shadow of Object = _____ (ft. and in.) = _____ (in.)



Students will then have to use similar triangles (ratios) to estimate the height of the object.

$$\frac{\text{Height}_{\text{person}}}{\text{Shadow}_{\text{person}}} = \frac{\text{Height}_{\text{object}}}{\text{Shadow}_{\text{object}}}$$

Method 2) Students will determine and measure how far back they have to stand to cover the object with one Māhā or Kaulua (these units work best with large objects. If measuring smaller objects, use smaller measurements).

Height of Person = _____ (ft. and in.) = _____ (in.)
 Shadow of Person = _____ (ft. and in.) = _____ (in.)
 Shadow of Object = _____ (ft. and in.) = _____ (in.)

DAY 2: COLLECTION OF DATA, ESTIMATION, AND DISCUSSION

- If not done on day 1, spend time discussing how hand measurements and shadows can be used to estimate the heights of objects.
- Split the class into groups and then collect data on two objects and record it on the Hawaiian Anthropic Measurements worksheet. You will need tape measures and a sunny day. Preferably 90 minutes before or after solar noon (60 minutes is okay). Object 1 should be the same for all groups and should be something that cannot be directly measured (flagpole, telephone pole, building, tree). Object 2 should be different for each group and should be something that can be measured directly so that estimations can be compared.
- Have students write on the board the height estimate they calculated for Object 1 using their shadows. As a class, discuss how they will come up with a class estimate. Why are there differences in estimations?

4. Which method seems more accurate, shadow or hand? How could the estimate be improved?
5. Object 2 allows students to compare hand measurements with shadow measurements. As a class, discuss if the estimates were similar to the actual results. If they were different, were all groups over, under, or a mix? Did one method (shadow or hand) seem more accurate?

REFERENCES/RESOURCES:

Dey, D. (2016, January). E HUKI I KE KAULA: Pull the rope, reach for the horizon. Unpublished internal document, Ethnomathematics and STEM Institute.

Hawaii State Department of Education (2015, November). Nā Hopena A’o (“HĀ”) Statements. Retrieved from <http://www.hawaiipublicschools.org/DOE%20Forms/NaHopenaAoE3.pdf>

Nā Honua Mauli Ola—Cultural Pathways for Culturally Healthy and Responsive Learning Environments. Retrieved from <http://www.olelo.hawaii.edu/pdf/NHMO-2014-Poster-Pelaha.pdf>.

NAME _____ PERIOD _____ DATE _____

HAND-TO-DEGREE CALIBRATION CALCULATION WORKSHEET

DEGREE	TOTAL	TOTAL	_ft _in		Approx _ft _in	
	WALL SIDE (FT)	INCHES	ft	in	ft	in
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						

NAME _____ PERIOD _____ DATE _____

ESTIMATION WITH HAWAIIAN ANTHROPIC MEASUREMENTS

Part A) Get this data from the whiteboard in the classroom.

Māhā = _____ degrees

Kaulua = _____ degrees

Part B)

First Object (round answers to the nearest inch)

Height of Person = _____ (ft. and in.) = _____ (in.)

Shadow of Person = _____ (ft. and in.) = _____ (in.)

Shadow of Object 1 = _____ (ft. and in.) = _____ (in.)

Distance from Object 1 at (Māhā /Kaulua) = _____ (ft. and in.) = _____ (in.)

My estimate of Object 1 using shadows is _____

My estimate of Object 1 using my hand is _____

Actual height of Object 1 (if known) _____

Second Object (round answers to the nearest inch)

Height of Person = _____ (ft. and in.) = _____ (in.)

Shadow of Person = _____ (ft. and in.) = _____ (in.)

Shadow of Object 2 = _____ (ft. and in.) = _____ (in.)

Distance from Object 2 at (Māhā /Kaulua) = _____ (ft. and in.) = _____ (in.)

My estimate of Object 2 using shadows is _____

My estimate of Object 2 using my hand is _____

Actual height of Object 2 (if known) _____