Photo by Karen Blaha

Standard Benchmarks and Values

Common Core State Standards for Mathematics:

- **F.TF.5** - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- **F.IF.7** - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Common Core Mathematical Practices:

- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 7) Look for and make use of structure.

Next Generation Science Standard:

- **HS-PS4-1** Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

EXPLORING OCEAN WAVES WITH THE SINE CURVE

Laurie Schafer

10-12 (Trigonometry)
1.5 - 2 hours (additional exercises and extension may take longer)*

**Note: This time frame assumes students have a basic understanding of how to use the desmos.com graphing calculator.*

How is the sine function related to ocean waves?

- How do the a and b terms in $f(x) = a\sin(bx)$ relate to an actual ocean wave?
- How do changing the a and b terms affect the graph?
- Why is knowing the period of a wave useful to understanding the wave?

Enduring Understandings

- Students will create a wave in a tank to simulate an ocean wave and develop a sine curve to model the wave.
- Students will create their own definitions for amplitude and wavelength and explore these concepts.
- Students will further explore the concepts of period, frequency, and wave speed using real life data.

Critical Skills and Concepts

- Students will create a sine curve to model a wave.
- Students will adjust amplitude, midline, and wavelength to fit their curve.
- Students will compare/contrast different sine curves and equations.
- Students will develop a working definition for amplitude and wavelength.
- Students will examine the relationship between period and frequency.
- Students will analyze real life waves.

LESSON

Cultural Connection

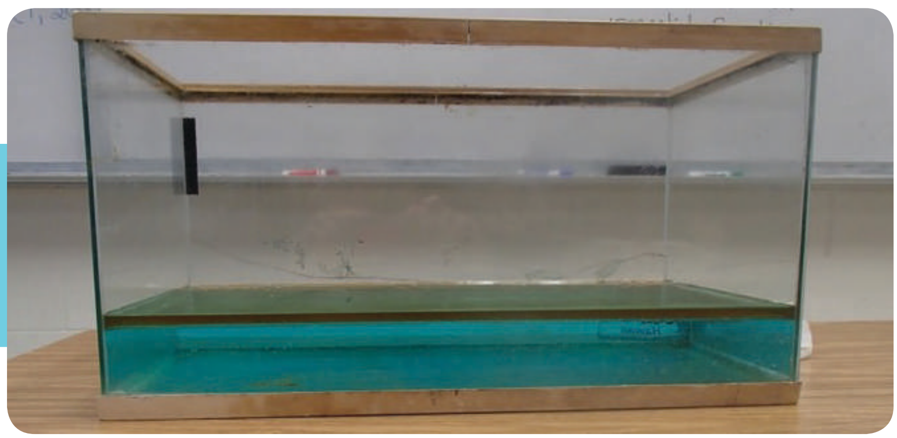
This lesson is developed for students that have a deep connection to the ocean such as those living in Hawaiʻi. Students may have already made some of these observations from their everyday experiences with the ocean but will now be given the vocabulary and context to discuss features of ocean waves.

Materials

- fish tank or clear plastic tub
- water to fill tank
- food coloring for water (optional)
- oil (optional)
- digital camera or phone with camera capable of uploading pictures to a computer
- computer for each student group with access to www.desmos.com calculator

Authentic Performance Task

1) Create wave tank.



Fish tank filled with water, food coloring and oil.

Photo by Laurie Schafer

- In student work groups, create a wave tank by filling an aquarium or clear plastic tub approximately a fourth of the way with water. (Optional: Add food coloring to the water and pour a layer of oil on top to see the contrast. The oil will help to create a smooth wave.)

2) Use the tank to make waves.

Student moves table to create waves inside fish tank.

Photo by Laurie Schafer

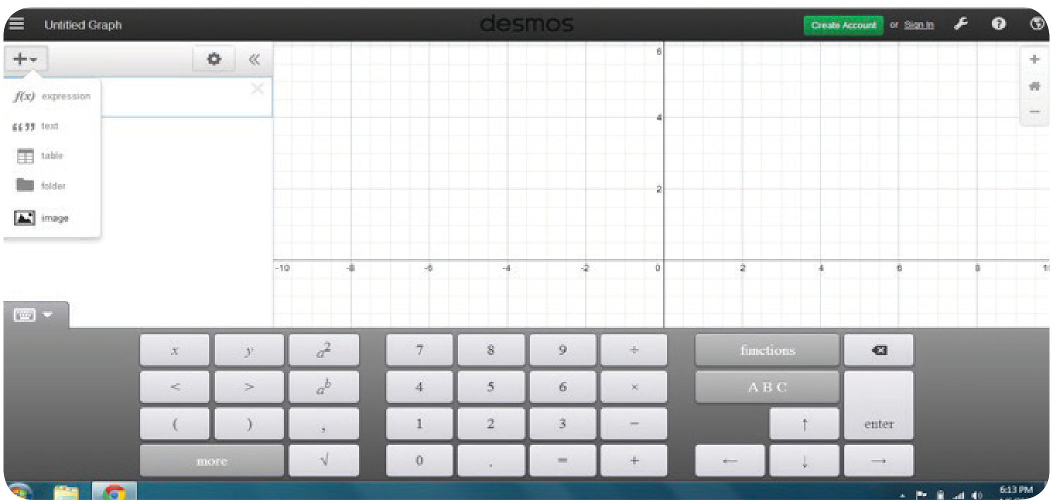
- Have students experiment by moving the table to create waves. Allow each group to create their own wave. They may choose to adjust the water level or speed of the wave. The more differences the better. The student in the picture is lifting the end of the table up and down to create a wave.

3) Take a picture.

Photo by Laurie Schafer

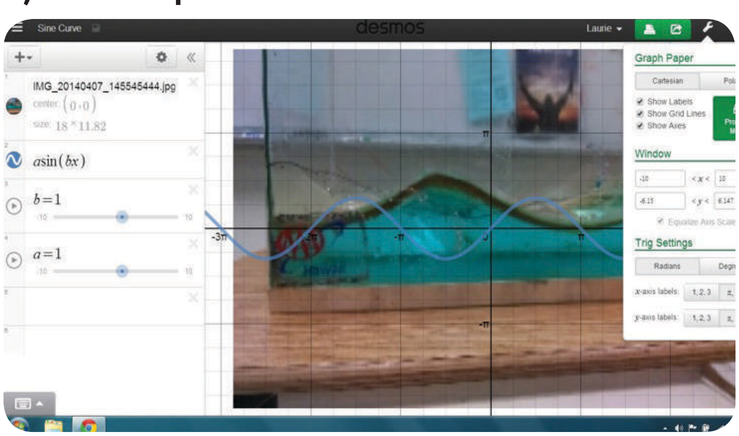
- Each group should take pictures of their wave from a side view until they get a clear shot of a steady wave.

4) Create a sine function to model curve.



- Upload the picture to a computer.
- Go to www.desmos.com/calculator and click on the add item menu (plus sign with drop down arrow at the top left of the screen). Select add image. Add the picture of the group's wave.

5) Create equation to match the wave.



- Select the "sin" function from the functions menu on the keyboard
- Change sin( ) to asin(bx). Click "add slider" for a and b. This allows students to change the a and b terms to match their wave. If the group's wave is not centered or the midline is off, they may have to change the center. Students can also change from degrees to radians using the settings menu as shown below. Allow students time to fit the equation to their wave.

DISCUSS

Discussion Questions

(These questions may be printed and given to students to answer individually or as a group or the teacher may choose to use them in a whole class discussion format.)

- Compare/contrast your equation and graph with that of another group. How do you relate the differences/similarities in equations to the differences/similarities in the graphs?
- How does changing the a term in f(x) = $a$sin(bx) affect the graph? The a term is called the amplitude of the function. Based on your observations, write a definition for the term amplitude.
- How does changing the b term in f(x) = $a$sin(bx) affect the graph? The b term is called the wavelength of the function. Based on your observations, write a definition for the term wavelength.
- Could this function be modeled with f(x) = $a$cos(bx)? Why or why not?
- Does your sine graph fit your equation perfectly? How can you account for any differences?
- If you wanted to increase your wavelength, what would you do differently to the wave tank?
- If you wanted to increase your amplitude, what would you do differently to the wave tank?

Rubric for Discussion Questions

Category	4	3	2	1
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR was not included.
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written

Other Related Concepts

- **<u>Period (*T*):</u>** the amount of time it takes for one wavelength to pass a specific point in space, measured in seconds.

$$T = \frac{2\pi}{\lambda} \text{ where } \lambda \text{ is wavelength}$$

- **<u>Frequency (*f*):</u>** the number of waves that pass a given point in a second, measured in Hz.

$$f = \frac{1}{T}$$

- **Wavelength** and **period** can be used to calculate a wave's <u>speed</u>.

$$speed\ (v) = \frac{wavelength\ (\lambda)}{period\ (T)} \text{ which is related to the known formula } v = \frac{distance}{time}$$

EXTRA

Additional Exercises

Applying a new vocabulary.

- Have students find the period of their wave, then calculate the frequency. Have students express their results in full sentences clarifying the meaning of each number.

  (For example: The amount of time it takes for one wavelength to pass a specific point is 5 seconds; therefore, .2 waves will pass a given point in a second.)

Using buoy data to make wave predictions.

- Go to www.hawaiisurfnews.com and select any of the buoys listed at the top of the page. This will give students current wave heights, dominant wave period, average wave period, and mean wave direction.
- From this data, students can calculate the frequency of the waves.
- Period (T)can be used to find wavelength (λ)

$$\lambda = \frac{gT^2}{2\pi} \text{ where } g \text{ is the acceleration due to gravity, } g = 9.8m/s^2 \text{ or } g = 32.2ft/s^2$$

- Students can then find the speed of the waves.
- Using these measures, have student compare/contrast the waves of two or more different buoys. Students should describe in detail the differences in the waves. Based the results, students may want to discuss which area would currently be better for surfing, boating, or a family day at the beach.

Extension - Tsunami Waves

- Research past tsunamis and allow students to analyze the data. Students can discuss how tsunami waves are different from the waves they analyzed using buoy data.

  http://www.ngdc.noaa.gov/hazard/tsu.shtml