Elementary (5th grade)

ORIGAMETRY (ORIGAMI GEOMETRY)

How does origami relate to two- and three-dimensional geometry?

by Shayla Theodore

Standard Benchmarks and Values:

Mathematics Common Core State Standards (CCSS):

• 5.MD.5b: Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

Hawaiʻi Content & Performance Standards (HCPS) III:

• MA.5.4.5: Use known measurements (e.g., base and height) to calculate desired measurements (e.g., area) of triangles, parallelograms, and trapezoids.

• MA.5.4.6: Use known measurements (e.g., length, width, and height) to calculate desired measurements (e.g., surface area and volume) of rectangular solids.

Nā Honua Mauli Ola (NHMO) Cultural Pathways:

• ʻIke Ola Pono (Wellness Pathway): Caring for the wellbeing of the spirit, naʻau, and body through culturally respectful ways that strengthen one's mauli and build responsibility for healthy lifestyles.
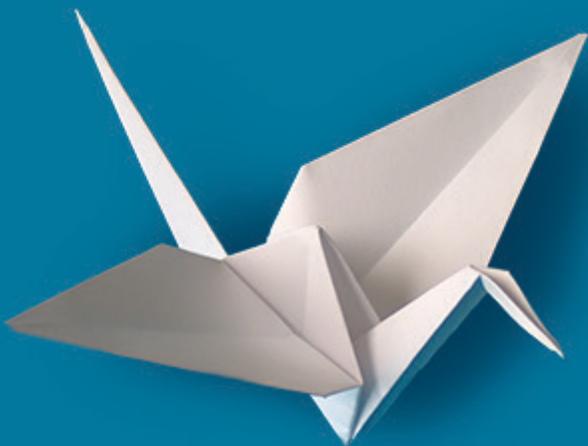
• 'Ike Piko'u (Personal Connection Pathway): Promoting personal growth, development, and self-worth to support a greater sense of belonging, compassion, and service toward one's self, family, and community.

• 'Ike Kuana'ike (Worldview Pathway): Providing a solid grounding in a Hawaiian worldview that promotes contributions to local and global communities.

Enduring Understandings:

• Origami is a *cultural* and *mathematical* tradition.

• Folding paper is a highly accessible, age-old pastime for people of all walks of life, regardless of nationality, socioeconomic level, age, gender, or religion.

Background/Historical Context:

Origami, Sadako, & Hawai'i

Origami is a traditional Japanese art form that entails folding a flat square of paper into a three-dimensional sculpture without the use of cutting or gluing, which would instead be *kirigami*. Although its practice dates back to 17th-century-A.D. Japan, paper folding has also been a tradition in other parts of Asia (e.g., China) and even in Europe (e.g., Germany, Italy, and Spain). However, Akira Yoshizawa's work in the early 1900s inspired a renaissance of the art form, and starting in the 1980s, mathematical studies of origami led to applications ranging from how stent implants are surgically inserted to repair blood vessels to how car airbags and even space satellite solar panels are deployed.

The paper crane (or *orizuru*) is one of the most popular origami sculptures, since, according to Japanese legend, whoever folds 1,000 cranes (collectively called *senbazuru*) will be granted a wish. The children's novel *Sadako and the Thousand Paper Cranes* (published by Eleanor Coerr in 1977) tells the story of a real girl named Sadako Sasaki, who lived in Hiroshima, Japan at the time of its atomic bombing by the United States on August 6, 1945 during World War II. When she was 11 years old, she learned that she had developed leukemia as a result of exposure to the bomb's radiation and, while in the hospital, aspired to fold 1,000 origami cranes in hopes of being granted a single, simple wish: to live. Today, Sadako is immortalized by statues within both the Hiroshima Peace Memorial Park and the Seattle Peace Park and serves not only as a symbol of the impact of nuclear war but also a symbol of peace.

After King Kalākaua visited Japan in 1881 on a peaceful mission to strengthen relations between his and Emperor Meiji Mutsuhito's nations, Japanese immigrants began arriving in 1885 to work as contract laborers for Hawai'i's sugar cane and pineapple plantations. Today, *nikkei* (or Japanese immigrants and their descendants) comprise approximately one fifth of Hawai'i's total population, and origami (especially paper crane folding) and other Japanese cultural traditions continue to be practiced and shared by many of Hawai'i's residents, regardless of ethnicity.

Authentic Performance Task:

Students will demonstrate how to fold a larger-scale origami crane and how to calculate its *surface area* and approximate *volume.*

Authentic Audience:

Fellow students, teachers, parents, and other family members.

Learning Plan:

1. Share the history of origami with students and give a synopsis of Sadako and the Thousand Paper Cranes (without revealing the ending). Show examples of the origami sculptures that the students will be constructing (i.e., a mountain, a box, a turtle, a cup, a balloon, and a crane).

2. Review with students how to calculate the following:

    -   Area of a triangle (i.e., a polygon with 3 sides):

    $$A = \tfrac{1}{2} \times base \times height = \tfrac{1}{2} bh$$

    -   Area of a *quadrilateral* (i.e., a polygon with 4 sides):

    $$A = base \times height = bh$$

    -   Area of a polygon with 5 or more sides:

        a. Divide the larger polygon into smaller quadrilaterals and triangles (that is, as few as possible but accounting for the original polygon's entire area).
        b. Calculate the individual areas of each triangle and quadrilateral and add these together for the polygon's total area.

    -   Surface area and volume of a rectangular prism:

    a. Calculate the area of each side ($A = bh$) and add these together for the total surface area;
    b. $V = length \times width \times height = l \times w \times h$

3. Pass out sheets of 8.5" x 11" paper, scissors, and rulers to each student. Have students fold and calculate the *area of* the following (and any other polygons formed during the folding process), measuring the proper lengths (with the ruler) and choosing the appropriate equation(s):

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Origami Mountain

Origami Open-box

Origami Cup

Origami Turtle

Images from www.wikihow.com

   a. *Origami mountain* (using the steps provided at http://www.wikihow.com/Make-an-Origami-Mountain);

   b. *Open box* (http://www.wikihow.com/Fold-a-Paper-Box);

   c. *A turtle* (http://www.wikihow.com/Make-an-Origami-Turtle);

   d. *A cup* (http://www.wikihow.com/Fold-a-Cup-from-a-Sheet-of-Paper);

4. Next, have students fold a balloon (http://www.wikihow.com/Make-an-Origami-Balloon) and calculate its surface area and volume.

5. Using poster paper, have students practice and eventually demonstrate how to perform the following:

   a. Fold a larger-scale paper crane (http://www.wikihow.com/Fold-a-Paper-Crane),

   b. Calculate its surface area (using a combination of the aforementioned equations), and

   c. Approximate its volume.

6. Extensions:

• Have the students read Sadako and the Thousand Paper Cranes, folding different origami shapes (http://www.wikihow.com/Category:Origami) to represent different aspects of the story. If folding cranes gave Sadako hope that she might get better, ask the students to consider and share what gives them hope when they are faced with challenges.

• As a class, fold 1,000 paper cranes to donate to a hospital, nursing home, or other worthy cause as a symbol of hope.

Origametry: Origami Geometry

Standard Benchmarks, GLOs, or Nā Honua Mauli Ola	Skills	Concepts	Assessment
MA.5.4.5: Use known measurements (e.g., base and height) to calculate desired measurements (e.g., area) of triangles, parallelograms, and trapezoids.	- Identifying and classifying two dimensional shapes. - Identifying the base and height of various polygons.	- Logic - Classification - Properties of shapes	- Accurate representation of key concepts using sketches, verbal descriptions, and visual models.
MA.5.4.6: Use known measurements (e.g., length, width, and height) to calculate desired measurements (e.g., surface area and volume) of rectangular solids.	- Recognizing how and where to measure length, width, and height.	- Measurement - Spatial reasoning	- Accurate and appropriate measurements. - Appropriate equation choices.
5.MD.5b: Apply the formulas V = l x w x h and V = b x h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.			

References:

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Coerr, E. (1977). Sadako and the thousand paper cranes. New York, NY: Penguin Group.

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